

Introduction to Deep Learning for Speech and Language Processing

Exercise Sheet 1: Math for Machine Learning

17th October 2025

Notation

We try our best to be consistent with the mathematical notation throughout the course. Here are the most important conventions we use:

Symbol	Example	Description
greek letter	α	scalar
lower-case letter	x	vector (column vector)
... with one subscript	x_i	i -th entry of vector x (scalar)
... with two subscripts	x_{ij}	entry in the i -th row, j -th column of a matrix (scalar)
upper-case letter	M	matrix
	M^{-1}	inverse matrix of matrix M
	v^\top, M^\top	transposition of a vector or matrix
	I_n	$n \times n$ identity matrix
	$x \cdot y$	dot product of two vectors

Linear Algebra

Vector Operations

Exercise 1.

Compute.

$$(1) \ y = \alpha x + b \text{ for } \alpha = \frac{1}{6}, x = \begin{bmatrix} -2 \\ 9 \\ 3 \\ -4 \end{bmatrix}, b = \begin{bmatrix} 1 \\ \frac{1}{2} \\ -\frac{1}{2} \\ -2 \end{bmatrix}$$

$$(2) \ z = x \cdot y = \langle x, y \rangle \text{ for } x = \begin{bmatrix} 1 \\ -2 \\ -\frac{1}{2} \\ 3 \end{bmatrix}, y = \begin{bmatrix} -3 \\ 7 \\ 2 \\ 2 \end{bmatrix}$$

$$(3) \ z = x \odot y \text{ for } x = \begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix}, y = \begin{bmatrix} 1 \\ 2 \\ 8 \end{bmatrix}$$

Vector Similarity

Exercise 2.

The following vectors represent word embeddings: $dog = \begin{bmatrix} 2.0 \\ 1.5 \\ 3.0 \end{bmatrix}$, $puppy = \begin{bmatrix} 3.1 \\ 1.25 \\ 4 \end{bmatrix}$, $car = \begin{bmatrix} -4.7 \\ 2.1 \\ -5.5 \end{bmatrix}$.

Compute the pair-wise cosine similarity between

- (1) dog and $puppy$
- (2) dog and car

Linear Independence

Matrix Operations

Exercise 3.

Compute.

(1) A^T for $A = \begin{bmatrix} 3 & 1 & 0 \\ -1 & -2 & 5 \end{bmatrix}$

(2) $Wx + b$ for $W = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -2 & \frac{1}{2} \end{bmatrix}$, $x = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$, $b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(3) $C = AB$ and $D = BA$ for $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -2 & \frac{1}{2} \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 0 & 1 \\ -2 & -1 \end{bmatrix}$

(4) $F = DE$ for $D = I_3$, $E = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$